A closer look at the Epps effect*

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Abstract

Epps (1979) reported empirical evidence that stock correlations decrease when sampling frequency increases. This phenomenon, named Epps effect, has been observed in several markets. In this paper, the dynamics underlying the Epps effect are investigated. Using Monte Carlo simulations and the analysis of high frequency foreign exchange rate and stock price data, it is shown that the Epps effect can largely be explained by two factors: the non-synchronicity of price observations and the existing lead-lag relationship between asset prices. In order to compute co-voltalities, an original method based upon the Fourier analysis is adopted. This method performs well in estimating correlations precisely, as illustrated by simulated experiments. Being naturally embedded in the frequency domain, this estimator is well suited to the study of the Epps effect.

JEL Classification: C32, G10, F30

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1 Introduction

In his 1979 paper, Epps reported empirical evidence of a dramatic drop in correlations between stocks when decreasing the sampling time horizon. This phenomenon has been observed across different markets, see for example Bonanno et al. (2001); Zebedee (2001) for stock prices, Lundin et al. (1999); Muthuswamy et al. (2001) for foreign exchange rates. On the other hand Andersen et al. (2001a) and Andersen et al. (2001b), with regard to stock prices and foreign exchange rates respectively, report correlations significantly different from zero when computed using five minute returns.

In this paper, as the title suggests, there is an attempt to understand the dynamics underlying the Epps effect better. For this purpose, we adopt the Fourier variance-covariance estimator set out in Malliavin and Mancino (2002), whose properties as a variance estimator have been studied in Barucci and Renò (2002a, b). This estimator has two appealing features: first, it uses all the tick-by-tick observations with no need to change their time structure. Second, all the other estimators developed so far (Andersen and Bollerslev, 1998; de Jong and Nijman, 1997; Lundin et al., 1999; Ball and Torous, 2000) are constructed in the time domain, while the Fourier estimator is naturally embedded in the frequency domain. The Epps effect deals with the behavior of correlations as a function of the sampling frequency, so the Fourier estimator is well suited for its inspection. In this paper, the Fourier estimator is used for the first time to estimate correlations.

The main idea pursued in this paper is that correlation measurements are always biased toward zero when observations are not perfectly synchronous. Two main statistical features of the data may produce this effect: asynchronous trading and lead-lag relationships. The impact of asynchronous data on covariance measurement has been widely studied: two examples are Lo and MacKinlay (1990) and, formerly, Scholes and Williams (1977). In addition to this, it is sometimes suggested that the Epps effect may depend upon the fact that correlations are lagged\(^1\), so that when reducing the sampling frequency under time scales comparable to this lag, the correlation measurements turn out to be lower.

The motivation of this work is to assess the following questions:

- what is the relative magnitude of the impact of asynchronous trading and lead-lag relationships?

\(^1\)It is worth to remark that non-synchronous trading itself could be a source of spurious lead-lag relations, see Chan (1992, 1993).
are these two features sufficient to explain the Epps effect?

In the literature, we have found mixed answers to these two questions. For example, Zebedee (2001) argues that the Epps effect is mainly due to the lead-lag relationship, maintaining that as frequency increases, correlation is shifting to other nearby time intervals. On the other hand, Lundin et al. (1999) claim that different actors play different roles at different frequencies, so that it is not possible to recover the same correlation at different time scales. However, it is not clear which kind of price formation process could lead to the Epps effect. Moreover, Lundin et al. (1999) find a significant inverse relation between correlation and activity: the more an asset is traded, less evident is the Epps effect. Again this seems to enforce the importance of synchronicity in explaining the correlation decrease at higher frequencies.

In this paper, we will primarily make use of Monte Carlo simulation, showing that, if price observations of two traded assets are synchronous, and if there is no lead-lag relationship, no frequency effects should be observed in the correlation measurements. Non-synchronicity and lags are then introduced, and the results show that both have a substantial effect (although it is mostly due to synchronicity), and thus are good candidates to explain the Epps effect. We then turn to the analysis of foreign exchange rates and stock prices, showing that, when applying adjustment techniques based only on these two factors, a significant attenuation of the drop in correlation measurement at high frequencies is observed.

The remainder of the paper is organized as follows: Section 2 briefly describes the Fourier estimator and its implementation. Section 3 analyzes the Epps effect on Monte Carlo experiments. Results for foreign exchange rates and stock prices are given in Section 4. Conclusions are presented in Section 5.

2 The Fourier variance-covariance estimator: theory and implementation.

The Fourier estimator for the continuous-time variance-covariance matrix has been suggested in Malliavin and Mancino (2002). Given \( p_j(t), j = 1, \ldots, D \), the \( R^D \)-valued stochastic process of interest, its covariance matrix is defined as:

\[
\Sigma_{ij}(t) := \lim_{\epsilon \to 0} \frac{1}{\epsilon} \mathbb{E} \left[ (p_i(t + \epsilon) - p_i(t)) \cdot (p_j(t + \epsilon) - p_j(t)) \right] |\mathcal{F}_t|, \quad i, j = 1, \ldots, D, \tag{1}
\]
where $\mathcal{F}_t$ denotes the filtration at time $t$, see Malliavin (1997) for further details.

For further analysis $\Sigma_{ij}$ is required to be well-defined (i.e. the above limit does exist) and bounded for every $t$. This is the case for stochastic volatility models of the kind:

$$dp_i(t) = \sum_{j=1}^{D} \sigma_{ij}(t)dW_j(t) + \mu_i(t)dt \quad i = 1, \ldots, D,$$

where $\sigma : R \rightarrow R^{D \times D}, \mu : R \rightarrow R^D$ are time dependent random functions and $W : R \rightarrow R^D$ is a vector of independent Brownian motions. If we denote with $S_t(t)$ a vector of generic asset prices, we will set $p_i(t) = \log S_t(t)$. It is straightforward to show that, for models like (2), the covariance matrix (1) is given by

$$\Sigma_{ij}(t) = \sum_{k=1}^{D} \sigma_{ik}(t)\sigma_{kj}(t).$$

The idea behind the Fourier estimator is to compute the Fourier coefficients of $\Sigma_{ij}$ from the Fourier coefficients of $dp_i$. For simplicity’s sake, the time interval into which the time series is recorded is compressed to $[0, 2\pi]$. The Fourier coefficients of $dp_i$ are defined in the usual way:

$$a_0(dpi) = \frac{1}{2\pi} \int_0^{2\pi} dp_i(t)$$

$$a_k(dpi) = \frac{1}{\pi} \int_0^{2\pi} \cos(kt)dp_i(t)$$

$$b_k(dpi) = \frac{1}{\pi} \int_0^{2\pi} \sin(kt)dp_i(t),$$

and similar formulas hold for $a_k(\Sigma_{ij}), b_k(\Sigma_{ij})$; from the Fourier coefficients of $\Sigma_{ij}$, $\Sigma_{ij}(t)$ can be obtained pointwise by the Fourier-Fejer inversion formula:

$$\Sigma_{ij}(t) = \lim_{n \to \infty} \sum_{k=0}^{n} \left(1 - \frac{k}{n}\right) \cdot [a_k(\Sigma_{ij}) \cos(kt) + b_k(\Sigma_{ij}) \sin(kt)].$$

In Malliavin and Mancino (2002) it is proved that:

$$a_k(\Sigma_{ij}) = \lim_{M \to \infty} \frac{\pi}{M + 1 - n_0} \cdot \sum_{s=n_0}^{M} \frac{1}{2} [a_s(dp_i) a_{s+k}(dp_j) + a_s(dp_j) a_{s+k}(dp_i)],$$

where $n_0$ is a given integer. Thus, formula (6) allows to compute the Fourier coefficients of $\Sigma_{ij}$ from the Fourier coefficients of $dp_i$, then to reconstruct $\Sigma_{ij}(t)$ via (5). However, we are
not interested in the pointwise covariance matrix $\Sigma_{ij}$, but in its integrated value over the time window, defined as:

$$\hat{\sigma}_{ij}^2 = \int_0^{2\pi} \Sigma_{ij}(s) ds. \quad (7)$$

In our framework, this is easily given by:

$$\hat{\sigma}_{ij}^2 = 2\pi a_0(\Sigma_{ij}), \quad (8)$$

where $a_0(\Sigma_{ij})$ is given by (6).

This estimator is then implemented as follows: since it is not possible to compute directly the Fourier coefficients of $dp_t$, these coefficients are computed via integration by parts:

$$a_k(dp_t) = \frac{1}{\pi} \int_0^{2\pi} \cos(kt) dp_t(t) = \frac{p(2\pi) - p(0)}{\pi} - \frac{k}{\pi} \int_0^{2\pi} \sin(kt)p_t(t) dt. \quad (9)$$

In financial markets, $p_t(t)$ is not observed continuously, but it is unevenly sampled in the form of tick-by-tick observations, $p_t(t_k), k = 1, \ldots, N$. Thus, we need to make an assumption on the interpolation of prices when computing the integrals in (4); we use $p(t) = p(t_j)$ where $t_j$ is the largest observation time before $t$. Usually, this interpolation scheme is referred to as the previous-tick interpolation; see Barucci and Renò (2002a); Corsi et al. (2001) for a discussion of this point. For all computations, we set $n_0 = 1$.

A crucial point here is the choice of the maximal $M$ in the expansion (6). A feature which is commonly displayed in high frequency data is that continuous-time models are not flexible enough at very high frequencies, where microstructure effects would be better described by tick-by-tick models. However, as shown in Barucci and Renò (2002a), this empirical feature does not necessarily affect the multivariate volatility estimate. Indeed, the expansion (6) can be stopped at a proper frequency, in order to rule out microstructure effects. The choice of the stopping frequency $M$ is largely an empirical matter, but it can be easily inferred by looking at the volatility spectrum\(^2\). In this paper, the results of Barucci and Renò (2002a), who analyze the same data set used in this paper, will be adopted.

### 3 Monte Carlo experiments

The purpose of this paper is to analyze the behavior of correlations between high frequency asset prices, as a function of the sampling frequency. We first start with studying Monte

\(^2\)These plots are sometimes called “volatility signature plots” in the literature, see Andersen et al. (2000).
Carlo experiments. The simulation of two correlated asset price diffusions is accomplished via a bivariate continuous-time GARCH(1,1) model:

\[

dp_1(t) = \sigma_1(t) dW_1(t),
\]

\[
dp_2(t) = \sigma_2(t) dW_2(t),
\]

\[
d\sigma_1^2(t) = \lambda_1 [\omega_1 - \sigma_1^2(t)] dt + \sqrt{2\lambda_1 \theta_1} \sigma_1^2(t) dW_3(t),
\]

\[
d\sigma_2^2(t) = \lambda_2 [\omega_2 - \sigma_2^2(t)] dt + \sqrt{2\lambda_2 \theta_2} \sigma_2^2(t) dW_4(t),
\]

\[
corr(dW_1, dW_2) = \rho. \tag{10}
\]

All the other correlations between the Brownian motions are equal to zero. This is the bivariate continuous-time limit of the popular GARCH(1,1) model of Bollerslev (1986), see Nelson (1990); Drost and Werker (1996). This model has been implemented, for example, in Kroner and Ng (1998) on weekly stock prices. The choice of this model is motivated by the fact that it provides a reasonable description of the foreign exchange rate data which are going to be considered in the next section, see Andersen and Bollerslev (1998). In the same spirit, the parameters estimates on DEM-USD and JPY-USD as obtained in Andersen and Bollerslev (1998) will be used when simulating (10), i.e. \( \theta_1 = 0.035, \omega_1 = 0.636, \lambda_1 = 0.296, \theta_2 = 0.054, \omega_2 = 0.476, \lambda_2 = 0.480 \). It is worth noting that these estimates have been obtained independently from the two time series and that they are used here only for illustrative purposes. The value of the correlation is set to \( \rho = 0.35 \).

If we could observe the process (10) continuously, we would not have problems in computing the variance-covariance matrix and recover the exact value of the correlation. Here we want to study the impact of two main features of high-frequency data:

- intraday asset prices are recorded in form of tick-by-tick transactions or quotes, which are unevenly spaced and whose frequency depends on the liquidity of the asset;

- correlation may be lagged, due to different liquidity, economic significance or recording effects.

Using the Monte Carlo simulation, we should be able to disentangle the impact of these two effects on correlation measurements. We will proceed as follows: first, the bivariate process (10) is discretized using a first-order Euler discretization scheme with a time step of one second. This mimics continuous time synchronous record. Then, two kind of samples are simulated: in the first one, tick-by-tick durations are extracted from an exponential distribution with mean equal to 15 seconds for the first time series and to 45 seconds for
the second time series. The choice of the average duration again comes from the average durations of DEM-USD and JPY-USD respectively. In order to illustrate the effect of non-synchronicity of high frequency data, we use a second kind of sample in which the durations of both the time series are extracted from an exponential distribution with mean equal to 45 seconds, thus forcing the first time series to be observed exactly at the same times as the second time series. These two simulated time series are labeled asynchronous and synchronous respectively.

In order to introduce lagging, we use the sample described above, but shift all the observation times of the second time series backwards by 8 seconds, a choice motivated by subsequent data analysis. These two new samples are labeled synchronous-lagged and asynchronous-lagged respectively.

In these Monte Carlo experiments, daily (86400 seconds) covariance matrices are computed according to (8), as a function of frequency. Figure 1 shows the resulting average daily realized correlation as a function of the sampling frequency $M$ used in the computation, for the four different samples. Let us look at the asynchronous-lagged sample first, which is thought to be closer to actual data. Even if we do not go in the deep high frequency regime, for further comparison with FX data, the Epps effect is clearly displayed. Correlation begins to drop above a certain frequency, going far from the “true” generated value. If we increase the sampling frequency, then correlation goes to zero (not displayed). If we look at asynchronous (not lagged) data, we see that the Epps effect is still present, but slightly less relevant. It is evident that, in this Monte Carlo setting, microstructure effects are not present, so we can see that asynchronous quoting can be a very important factor explaining the reduction of the correlation measurements by itself. The effect of non-synchronicity on the correlation estimates of daily stock prices, which are typically recorded at slightly different closing hours, has been analyzed by Burns et al. (1998); Martens and Poon (2001) among others; these studies also show that this effect, which may look negligible at first glance, can seriously affect the correlation estimates.

We now turn to the analysis of synchronous data. From Figure 1, we observe that the Epps effect is dramatically reduced. However, we find another relevant source of the Epps effect. When we consider lagged data, we still observe a substantial drop in correlation, even if it appears to be smaller than that caused by non-synchronicity. The effect is visible even if the introduced lag, i.e. 8 seconds, is very small, at least when compared with the frequency $M = 160$, which corresponds to 3.5 minutes in the time domain.
Figure 1: Average correlation between two Monte Carlo simulations of asset prices, according to (10), as a function of the sampling frequency $M$ in (6). Boxes: simulated observation times are drawn independently. Asterisks: simulated observation times are drawn independently and the correlation is lagged by 8 seconds. Triangles: simulated observation times are forced to be the same and the correlation is lagged by 8 seconds. Circles: simulated observation times are forced to be the same, no lag. The generated value of the correlation is $\rho = 0.35$, corresponding to the dashed line in the figure. Error bars are computed according to the Normal distribution. These results are obtained with 10,000 replications.

In contrast, if data are synchronous and not lagged, we clearly observe that correlations do not drop at higher frequencies. Moreover, as expected, by increasing the sampling frequency we increase the measurement precision. To give a quantitative idea of the magnitude of this effect, we should link the correlation drop to the level of non-synchronicity or lag. Monte Carlo experiments were repeated keeping fixed the average duration of the second time series, set to $\tau_2 = 45$ seconds, and varying the average duration of the first, named $\tau_1$. Clearly, increasing $\tau_1$ will decrease the activity of the first time series, and consequently the number of synchronous quotes. Table 1 shows the results for the frequencies $M = 100, 130, 160$. Not surprisingly, we confirm the inverse relation between activity and correlation drop found in Lundin et al. (1999). We repeat these experiments by keeping fixed
Table 1: Correlation estimates on Monte Carlo experiments with asynchronous data. The second time series observations are extracted with an average duration of 45 seconds, while we report estimates at different values of the average duration $\tau_1$ of the first time series, at three different frequencies. All estimates are obtained with 1,000 replications.

<table>
<thead>
<tr>
<th>$\tau_1 \rightarrow$</th>
<th>15</th>
<th>25</th>
<th>35</th>
<th>45</th>
<th>55</th>
<th>65</th>
<th>75</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M = 100$</td>
<td>0.3452</td>
<td>0.3452</td>
<td>0.3437</td>
<td>0.3417</td>
<td>0.3384</td>
<td>0.3345</td>
<td>0.3297</td>
</tr>
<tr>
<td>$M = 130$</td>
<td>0.3373</td>
<td>0.3371</td>
<td>0.3354</td>
<td>0.3323</td>
<td>0.3279</td>
<td>0.3227</td>
<td>0.3162</td>
</tr>
<tr>
<td>$M = 160$</td>
<td>0.3311</td>
<td>0.3312</td>
<td>0.3288</td>
<td>0.3244</td>
<td>0.3184</td>
<td>0.3110</td>
<td>0.3028</td>
</tr>
</tbody>
</table>

Table 2: Correlation estimates on Monte Carlo experiments with synchronous lagged data. Both the time series observations are extracted with an average duration of 45 seconds, while we report estimates at different values of backward lag (in seconds) for the second time series, at three different frequencies. All estimates are obtained with 1,000 replications.

<table>
<thead>
<tr>
<th>LAG $\rightarrow$</th>
<th>4</th>
<th>8</th>
<th>12</th>
<th>16</th>
<th>20</th>
<th>24</th>
<th>28</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M = 100$</td>
<td>0.3512</td>
<td>0.3496</td>
<td>0.3485</td>
<td>0.3476</td>
<td>0.3479</td>
<td>0.3456</td>
<td>0.3441</td>
</tr>
<tr>
<td>$M = 130$</td>
<td>0.3488</td>
<td>0.3465</td>
<td>0.3442</td>
<td>0.3435</td>
<td>0.3431</td>
<td>0.3399</td>
<td>0.3382</td>
</tr>
<tr>
<td>$M = 160$</td>
<td>0.3487</td>
<td>0.3449</td>
<td>0.3421</td>
<td>0.3398</td>
<td>0.3383</td>
<td>0.3343</td>
<td>0.3329</td>
</tr>
</tbody>
</table>

the durations of the two time series to 45 seconds, and backwards the second time series backward by a variable lag. Table 2 shows the results for different frequencies $M$, as above. As expected, we observe a larger drop in correspondence with a larger lag.

It is important to determine the significance of these results. If we look at the standard deviation of measurements across the Monte Carlo sample at $M = 160$, which provides a reasonable estimate of the error, we find that it is around 0.05 for all four data sample. With 10,000 replications, this corresponds to a standard deviation of the mean a hundred times smaller. Thus, on the basis of a simple variance ratio test for the mean at 95% level, when comparing the correlations between the synchronous (not lagged) sample and the two asynchronous (lagged and not lagged) samples, we reject equality for $M \geq 60$, while
when comparing the synchronous sample with the synchronous lagged, we reject equality for \( M \geq 100 \). Kolmogorov-Smirnoff tests of equality among the distributions were also performed\(^3\), and they confirmed these results.

Finally, it seems quite interesting that the variance of the measurements is nearly the same\(^4\) across the four samples. This finding indicates that synchronicity plays an important role also in the precision of the Fourier estimator. Indeed, switching from asynchronous to synchronous data, we reduce the number of observations of the first time series by one third, but the standard deviation of measurements remains nearly the same.

The question now becomes whether the Epps effect observed in the literature can be explained only by means of the non-synchronicity of quotes and lead-lag relationship. The following Section tries to address this point by looking at market data.

### 4 Data analysis

The first data sample under study is the very well known collection of DEM-USD and JPY-USD exchange rate quotes as they appeared on the Reuters screen from October, 1\(^{st}\) 1992 to September 30\(^{th}\) 1993. The price is defined as the bid-ask midpoint. This data set has been collected and distributed by Olsen & Associates, and it has been extensively studied in the high frequency data literature.

The analysis by Barucci and Renò (2002a) suggests that the highest \( M \) that can be used in (6), in order to prevent microstructure effects from distorting our results, is given by \( M = 500 \) for the DEM-USD time series and \( M = 160 \) for the JPY-USD time series. These values will be used when computing variances, while when computing covariances, since the two time series are analyzed jointly, \( M \leq 160 \) will be always used. This frequency corresponds to roughly 3.5 minutes in the time domain. Figure 2 shows the average daily\(^5\) correlation as a function of the sampling frequency. Clearly the Epps effect is again in action, shifting the correlations downwards above a given frequency. In order to investigate the effect of non-synchronicity, our measurements are repeated using only synchronous data, i.e. quotes which have the same time stamp. This kind of data are about 16% of DEM-USD quotes and

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\(^3\)Detailed results on the tests are available from the author upon request.

\(^4\)The standard deviation at \( M = 160 \) is 0.0519 for asynchronous data, 0.0525 for synchronous data, 0.0522 for asynchronous lagged data, 0.0530 for synchronous lagged data.

\(^5\)We define one day starts and ends at 21:00 GMT.
42% of JPY-USD quotes, so we have a substantial reduction of our data sample. Results are again plotted in Figure 2. The reader may be confused by the different levels of correlation measurements in the different cases. These are due to the statistical fluctuations induced by the scarcity of observations, especially when using only synchronous data. The standard deviation of measurements ranges from 0.14 for asynchronous data to 0.18 for synchronous lagged data, thus, with 256 data points, standard error estimates are around 0.01. Thus the observed rise of correlation measurements when only synchronous observations are taken in

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6Actually, the number of synchronous quotes is surprisingly high. Indeed, we have on average 5653 DEM-USD quotes per day, and 2186 JPY-USD quotes per day. Taking into account that in our data sample quote times are rounded to the nearest even second, and assuming independent quoting, we expect on average 286 synchronous quotes per day, while we find 909! Clearly the assumption of independence is violated. For example, it is plausible that market makers post all their quotes contemporaneously.
Figure 3: Shows \([\rho(M) - \rho(64)]/\rho(64)\) as a function of \(M\), where \(M\) is the sampling frequency in (6). Crosses: all observations are included in the computation. Triangles: Only synchronous observations are included in the computation. Boxes: all observations are included, JPY-USD time series is shifted backwards by 8 seconds. Circles: Only synchronous observations are included in the computation after shifting the JPY-USD time series backwards by 8 seconds.

account is not statistically significant. Figure 2 is redrawn in Figure 3 in a different fashion: while Figure 2 plots \(\rho(M)\), Figure 3 plots \([\rho(M) - \rho(64)]/\rho(64)\), where \(M\) is the sampling frequency. We choose \(\rho(64)\) to normalize since it is the highest measured correlation in all the four cases.

We observe that the effect of correlation reduction when the sampling frequency increases is less strong. However, this effect is not completely canceled out. This result shows that the non-synchronicity of quotes plays a substantial role in the Epps effect, but other effects must be taken into account when trying to explain it.

The presence of lead-lag effect in our exchange rate data sample is checked by computing the daily lagged correlation:

\[
\rho(\tau) = \int_0^{2\pi} \text{corr} \left[p_1(t) p_2(t + \tau)\right] dt,
\]

as a function of \(\tau\), where \(p_1\) and \(p_2\) denote DEM-USD and JPY-USD exchange rate respec-
Figure 4: Average daily lagged correlation (11) between JPY-USD and DEM-USD exchange rates, as a function of $\tau$; a negative $\tau$ indicates that JPY-USD leads DEM-USD.

tively, and the interval $[0, 2\pi]$ denotes one day. Similar studies on lead-lag correlation have been conducted, for example, by Chan (1992); Ballocchi et al. (1999); Muller et al. (1997), using a much longer time scales than that analyzed in this paper. In the present analysis, again the Fourier method was used when computing (11), by computing the correlation after shifting the second time series temporally by $\tau$. Covariance is computed at $M = 64$. Results are shown in Figure 4. In our data sample the JPY-USD exchange rate leads DEM-USD by nearly 8 seconds. This lead-lag relationship looks very small when compared to the time scale we are investigating ($M = 160$, corresponding to nearly 4 minutes), but, as shown with Monte Carlo experiments, this could be enough to explain the drop in correlation measurements. Given that the two time series are both very liquid, and that the lag is found to be of the same order of magnitude of the average durations, we argue that this lag is likely to be due to non-synchronicity effects only.

The measurement of daily correlation between DEM-USD and JPY-USD is then repeated after shifting the JPY-USD time series backwards by 8 seconds. These measurements are performed using all the observations and synchronous observations only (after shifting), as in the previous case. Results are shown in Figure 2: by removing the joint effect of lead-
lag relationship and non-synchronicity (white circles) a strong reduction of the Epps effect is obtained. This is in agreement with what we found on Monte Carlo experiments (see Figure 1) on simulated time series. The lead-lag relation causes the Epps effect, even if we repeat the experiment forcing the two time series to be observed at the same time.

Figure 3 shows more clearly that using synchronous quotes, and adding a lag of 8 seconds, the Epps effect is strongly reduced. Comparing the measurements at $M = 160$ with those at $M = 64$, we obtain a 12.9% drop if we do not apply any correction; a 12.7% drop when lagging backwards the DEM-USD time series; a 9.0% drop when using only synchronous observations and a 5.3% drop when using only synchronous observations after lagging backwards by 8 seconds the DEM-USD time series. This drop is found to be statistically significant. Equivalence tests on the mean of $[\rho(M) - \rho(64)]$ are performed across the samples: at 95% level, we reject the equality of the measurements on the synchronous-lagged sample for $M \geq 124$ with the full sample, for $M \geq 131$ with the lagged sample, and for $M \geq 156$ with the synchronous (not lagged) sample. In order to further explain that 5.3% drop, it seems that other factors should be taken into account. The failure of continuous-time models to describe high-frequency data is the first factor under suspicion. However, a larger data sample should be used to understand these second-order effects.

All these results can be checked by repeating all the measurements on two much less frequently traded asset prices. The second data set under study consists of the high frequency trades of the stock prices of Exxon and Mobil, from January 1995 to April 1995 for a total of 82 trading days. Only trades from 9:30 to 16:00 at the NYSE time were used. We have on average 397 Mobil trades and 724 Exxon trades per day. These two stocks belong to the same economic sector (oil) and were eventually merged into the same firm on November 30th 1999, so there are good reasons to suppose that a substantial correlation should exist between the two time series considered. We start by checking the presence of a lead-lag effect of the same kind of that observed on FX rates. We use $M = 20$ when computing the variances of both stock prices, and $M = 10$ when computing the covariance.

Figure 5 shows that the lead-lag effect is again present, and in a more substantial way: the Mobil price leads Exxon by roughly a couple of minutes. On the basis of these results, we performed the correlation measurements on the raw data, on synchronous data only and on the data sample and only synchronous data after shifting the Exxon time series backwards by 70 seconds, corresponding to the largest correlation in figure 5. Figure 6 presents the results. We notice immediately that large fluctuations among measurements are
Figure 5: Average daily lagged correlation (11) between Mobil and Exxon stock prices, as a function of $\tau$; a positive $\tau$ indicates that Mobil leads Exxon.

present, due to the fact that the number of trades (and especially of synchronous trades) is considerably smaller than exchange rate quotes. We have on average 20 synchronous trades per day, and 12 synchronous trades per day after shifting. The standard deviation of the means is estimated around 0.02 for asynchronous data, and 0.04 for synchronous data, both lagged and not lagged. The Epps effect is again evident when computing correlations with all the trades, and again using only synchronous trades drastically reduces the effect. When using synchronous trades after shifting backwards the Exxon time series, the Epps effect is no longer observed, and correlations remain stable in the range $10 \leq M \leq 20$. Thus, the analysis of stock prices confirms the results obtained with FX rates.

These results can be very important from the risk management point of view. It is clear that a correct assessment of risk management strategies has to take care of correlations, and especially of the stochastic nature of correlations (Ball and Torous, 2000). Our results suggest that, even if continuous-time multivariate models fail to describe the dynamics of asset prices at the very high frequencies, it is still possible to use them to compute variances and correlations precisely with high frequency data, just taking into account non-synchronicity and lead-lag relationships. From this perspective, the Fourier estimator turns out to be a
Figure 6: Average realized daily correlation between Exxon and Mobil stock prices, from January 1st 1995 to April 30th 1995, as a function of the sampling frequency $M$ in (6). Crosses: all observations are included in the computation. Triangles: Only synchronous observations are included in the computation. Boxes: all observations are included, Exxon time series is shifted backwards by 70 seconds. Circles: Only synchronous observations are included in the computation after shifting the Exxon time series backwards by 70 seconds.

promising tool.

5 Conclusions

In this paper an investigation of the so-called Epps effect (Epps, 1979) is attempted. For this purpose, an original covariance estimator has been adopted, the Fourier estimator presented in Malliavin and Mancino (2002), and used for the first time in this paper to compute cross-correlations. This estimator is well suited to the time structure of high frequency data and to our frequency analysis. When tested on Monte Carlo bivariate experiments, the Fourier estimator proves to be a good candidate for computing correlations in a precise way. In our Monte Carlo experiments, price diffusions are simulated by a continuous-time model, namely the GARCH(1,1) continuous-time model. The results show that the Epps effect may
be explained by the non-synchronicity of quotes and by lead-lag relationships.

Further evidence for the Epps effect is derived from foreign exchange data. When considering only synchronous quotes, the effect is reduced but not eliminated. It is shown that in our sample there is a lead-lag relationship of 8 seconds. However, this very short time scale is enough to generate a frequency effect on correlations. If the measurements are performed using synchronous quotes after shifting one time series by 8 seconds, the Epps effect is drastically reduced. All these results are confirmed by the analysis of stock price data, where the lag is found to be around 70 seconds.

We conclude that even if other factors, apart from non-synchronicity and lead-lag relationships, which clearly play the main role, concur in the Epps effect, their effect is negligible. It remains questionable if these factors can be explained in the framework of continuous-time models.

It would be also interesting to find precise laws which relate the magnitude of the Epps effect to, say, correlation, average durations, lag, cut frequency. A Monte Carlo approach would turn out to be too time-consuming, given the number of parameters which affect the correlation estimates, and an analytic answer, as in Scholes and Williams (1977), would be desirable. We leave this topic for future research.

References


