THE RELATIONSHIP BETWEEN PUT AND CALL OPTION PRICES: COMMENT

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IN HIS ARTICLE on the relationship between put and call option prices, Stoll [4, p. 805-806] asserts that the forces of arbitrage (instituted by converters) will ensure an exact functional relationship between the value of a call option and the value of a put option: namely,[1]

$$C - P = V - E/(1 + i), \tag{1}$$

where C = value of a call with exercise price E and T-periods before expiration
P = value of a put with the same exercise price and maturity date
V = current value of the common stock
i = i(T), the risk-less, T-period rate of interest.

Stoll's derivation of (1) is correct under the assumption that, with (ex-ante) certainty, neither contract will be exercised prior to maturity.[2] Hence, (1) is only valid for "European"-type options which cannot be exercised prior to maturity. It does not follow that (1) will hold for the more common, "American"-type options where the option holder has the right to exercise prematurely. For (1) to obtain for American options, it must be shown that to exercise an option prematurely is never a rational policy which would imply that the right to do so has zero value (i.e., that American options should have the same value as their European counterparts). Although Stoll [4, p. 808-809] claims to prove this, his claim is incorrect.

While it has been shown for dividend-protected call options that premature exercising is irrational,[3] no such result holds for put options. A hint that no such theorem exists follows from first, noting that the value of an American put option must be a non-decreasing function of the length of time until maturity, and second, that the value of an European put option can be no greater than the present discounted value of its exercise price, $E/(1 + i)$.[4] For positive interest rates, $E/(1 + i)$ tends to zero as the time to expiration, T, goes to infinity. If the value of an American put option always equals the value of its European counterpart, then the value of the American put option must tend to zero as its time to maturity tends to infinity. But, the value of an American put option is a non-decreasing function of its time until expiration, from which it follows that all American (and hence, European) put options must have zero value which is clearly nonsense.

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1. Our equation (1) differs from Stoll's (5) only because he considered the case where V = E.

2. Additional assumptions required include equality of the borrowing and lending rates, use of proceeds of short-sales, and dividend protection. See [2] for a general discussion.

3. See [2] and [3] for proofs. In [2], it is shown that the standard provision for protecting call options against cash dividends is incorrect.

4. Since one can do anything with a longer-lived American option that can be done with a shorter-lived one, it cannot be worth less. The maximum pay-off to a put is the exercise price, E, which occurs if the stock price is zero. In the European case, the pay-off will not be received until the option expires. A security cannot sell for more than the present value of a sure payment of its maximum pay-off. See [2] for a general discussion.

A rigorous demonstration is as follows: let P_A = price of the American put option and P = price of its European counterpart. It follows from arbitrage that

$$P_A \geqq \mathrm{Max}[O, E - V]. \tag{2}$$

From (1), we have that $P_A = C - V + E/(1+i) < E - V$, if $C < iE(1+i)$. which is certainly possible for small enough V, since $C < V$. But, this would violate arbitrage condition (2), since the put owner would be better off to exercise his option immediately rather than hold it. Hence, $P_A > P$ if there exists a positive probability that sometime during the life of the contract, $C < iE/(1+i)$. But $P_A > P$ only if there is a positive probability of exercising prematurely. Thus, Stoll's assertion is disproved. The best one can do without a formal valuation theory are the bounding inequalities.

$$E \geqq P_A - C + V \geqq E/(1+i). \tag{3}$$

The essence of Stoll's theory is that conversion is risk-less and (1) is a direct result of that assumption. The previous analysis shows that conversion is not. Consider a converter who purchases a put, a long position in the stock, and sells a call. If the put were of the European type, the conversion would be risk-less and (1) would hold. However, if the put is American, the converter must pay a premium $P_A - P > 0$, for the right to exercise it prematurely, whether he wants it or not. If the *ex-post* pattern of stock price changes is such that the converter does not exercise his put option prior to expiration, then he will have a loss of $(1+i)(P_A - P)$,[5] independent of the terminal level of the common stock. Of course, under different ex-post price patterns where he exercises the put prematurely, he will show a gain. The point is that the return to the converter is risky with some chance of a loss.[6]

How likely is it that the value of a call option, initially written at market for six months, will have a value less than $iE/(1+i)$ prior to expiration? As a partial answer, consider the following example: let the original market value of the common be $100 per share and the value of a call option on 100 shares at market for six months be $1,300. Then, at an annual rate of interest of seven percent and using the rule of thumb that the call value declines half a point for each point decline in the common, we find that a drop in the stock price to about $80 per share would satisfy the inequality. More careful analysis, using the Black-Scholes [1] formula for call options, suggests that a decline to about $85 per share would be sufficient with the percent decline required smaller, the closer to the maturity date. Hence, one can conclude that rational premature exercising of puts is not only theoretically possible, but could also be expected to represent a significant fraction of all puts exercised.

REFERENCES

1. F. Black and M. Scholes. "Capital Market Equilibrium and the Pricing of Corporate Liabilities," forthcoming in *Journal of Political Economy.*
2. R. C. Merton. "Theory of Rational Option Pricing," *The Bell Journal of Economics and Management Science,* Vol. 4, No. 1 (Spring, 1973).
3. P. A. Samuelson and R. C. Merton. "A Complete Model of Warrant Pricing that Maximizes Utility," *Industrial Management Review,* Vol. 10, No. 2. (Winter, 1969), pp. 17-46.
4. H. R. Stoll. "The Relationship Between Put and Call Option Prices," *Journal of Finance,* Vol. XXIV, No. 5 (December, 1969), pp. 802-824.

5. This assumes that the converter can still borrow at the risk-less rate.

6. Careful analysis would show that the returns on the converter's position will be negatively correlated with returns on the common stock.

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[Footnotes]

[2] **Theory of Rational Option Pricing**
Robert C. Merton
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[3] **Theory of Rational Option Pricing**
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[3] **Theory of Rational Option Pricing**
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[2] **Theory of Rational Option Pricing**
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[4] **The Relationship Between Put and Call Option Prices**
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